Math 10 Lecture Videos

Section 4.3: Solving Systems of Linear Equations by the Addition Method

PAUL ANDREW GORGONIO

OBJECTIVES:

1. Solve linear systems by the addition method.
2. Use the addition method to identify systems with no solution or infinitely many solutions.
3. Determine the most efficient method for solving a linear system.

Objective 1: Solve linear systems by the addition method.

The third method we will consider for solving a system of equations is the **Addition Method**.

When we use this method, we will again try to eliminate one variable, but we will do that by adding the equations.

Addition method is also known as the **Elimination Method**.

NOTE: There is **more than one method** to solve a system of equations. The reason for learning more than one method is that sometimes one method will be **preferable** or **easier to use** over another method.

Objective 1: Solve linear systems by the addition method.

Solving Linear Systems by Addition
1. If necessary, rewrite both equations in the form $Ax + By = C$.
2. If necessary, multiply either equation or both equations by appropriate nonzero numbers so that the sum of the x-coefficients or the sum of the y-coefficients is 0.
3. Add the equations in step 2. *The sum should be an equation in one variable*.

Objective 1: Solve linear systems by the addition method.

Solving Linear Systems by Addition
4. Solve the equation in one variable (the result of step 3).
5. Back-substitute the value obtained in step 4 into either of the given equations and solve for the other variable.
6. Check the solution in *both* of the original equations.

Objective 1: Solve linear systems by the addition method.

Example 1: Solve by the Addition Method

$4x = -2y + 4$

$-y = -3x + 3$

1. Rewrite both equations in the form of *Ax + By = C*.

$4x + 2y = -2y + 4 + 2y$ Add 2y to both sides.

$4x + 2y = 4$

$-y + 3x = -3x + 3 + 3x$ Add 3x to both sides.

$3x - y = 3$

Objective 1: Solve linear systems by the addition method.

Example 1: Solve by the Addition Method

4x = -2y + 4

-y = -3x + 3

2. If necessary, multiply either equation or both equations by appropriate nonzero numbers so that the sum of the *x*-coefficients or the sum of the *y*-coefficients is 0.

$$4x + 2y = 4 \xrightarrow{\text{No Change}} 4x + 2y = 4$$

$$3x - y = 3 \xrightarrow{\text{Multiply by 2}} 6x - 2y = 6$$

Objective 1: Solve linear systems by the addition method.

Example 1: Solve by the Addition Method

4x = -2y + 4

-y = -3x + 3

3. Add the equations.

$$4x + 2y = 4$$

$$\underline{6x - 2y = 6}$$

Add: $10x + 0y = 10$

$$10x = 10$$

4. Solve the equation in one variable.

$$10x = 10$$

$$x = 1$$

Objective 1: Solve linear systems by the addition method.

Example 1: Solve by the Addition Method

4x = -2y + 4

-y = -3x + 3

5. Back-substitute and find the value of the other variable.

$$-y = -3x + 3$$

$$-y = -3(1) + 3$$ Replace x with 1.

$$-y = -3 + 3$$ Multiply

$$-y = 0$$ Multiply both sides by -1.

$$y = 0$$ Therefore, the *potential* solution is (1,0).

Objective 1: Solve linear systems by the addition method.

Example 1: Solve by the Addition Method

4x = -2y + 4

-y = -3x + 3

6. Check:

$$
\begin{aligned}
4x &= -2y + 4 \\
4(1) &\stackrel{?}{=} -2(0) + 4 \\
4 &\stackrel{?}{=} 0 + 4 \\
4 &= 4
\end{aligned}
\qquad
\begin{aligned}
-y &= -3x + 3 \\
-(0) &\stackrel{?}{=} -3(1) + 3 \\
0 &\stackrel{?}{=} -3 + 3 \\
0 &= 0
\end{aligned}
$$

Because *both* equations are satisfied, solution is (1,0).

Objective 1: Solve linear systems by the addition method.

Example 2: Solve by the Addition Method

$$3x - y = 27$$
$$4x + y = 8$$

1. Rewrite both equations in the form of $Ax + By = C$.

Not necessary, since the equations are already in Ax + By = C form.

2. If necessary, multiply either equation or both equations by appropriate nonzero numbers so that the sum of the *x*-coefficients or the sum of the *y*-coefficients is 0.

Not necessary here, since the sum of the *y*-coefficients is 0.

Objective 1: Solve linear systems by the addition method.

Example 2: Solve by the Addition Method

$3x - y = 27$

$4x + y = 8$

3. Add the equations.

$$\begin{array}{rl} 3x - y &= 27 \\ 4x + y &= 8 \\ \hline \text{Add: } 7x &= 35 \end{array}$$

4. Solve the equation in one variable.

$$\begin{aligned} 7x &= 35 \\ x &= 5 \end{aligned}$$

Objective 1: Solve linear systems by the addition method.

Example 2: Solve by the Addition Method

3x – y = 27

4x + y = 8

5. Back-substitute and find the value of the other variable.

$$\begin{aligned} 3(5) - y &= 27 \\ 15 - y &= 27 \\ -y &= 12 \\ y &= -12 \end{aligned}$$

Therefore, the *potential* solution is (5,-12).

Objective 1: Solve linear systems by the addition method.

Example 2: Solve by the Addition Method

$3x - y = 27$

$4x + y = 8$

6. Check:

$$\begin{aligned} 3(5)-(-12) &= 27 \\ 15+12 &= 27 \\ 27 &= 27 \end{aligned}$$

$$\begin{aligned} 4(5)+(-12) &= 8 \\ 20+(-12) &= 8 \\ 8 &= 8 \end{aligned}$$

Because *both* equations are satisfied, solution is (5, -12).

Objective 2: Use the addition method to identify systems with no solution or infinitely many solutions.

Solving Systems of Linear Equations

To determine that a system has **exactly one solution**, solve the system using one of the methods. A single solution will occur as in the previous examples.

To determine that a system has **no solution**, solve the system using one of the methods. Eventually, you'll get a *false statement*, like 3 = 4.

To determine that a system **has infinitely many solutions**, solve the system using one of the methods. Eventually, you'll get a *true statement*, like

Objective 2: Use the addition method to identify systems with no solution or infinitely many solutions.

Example 1. Solve:

$$x + 2y = 4$$
$$3x + 6y = 13$$

Multiply the first equation by −3 and then add.

$$\begin{aligned} -3x - 6y &= -12 \\ 3x + 6y &= \ \ 13 \\ \hline 0 &= \ \ 1, \text{ false} \end{aligned}$$

The false statement indicates that the system is inconsistent and has no solution.

Solution Set: { }

Objective 2: Use the addition method to identify systems with no solution or infinitely many solutions.

Example 2. Solve:

$$x - 5y = 7$$
$$3x - 15y = 21$$

Multiply the first equation by −3 and then add.

$$\begin{aligned} -3x + 15y &= -21 \\ 3x - 15y &= 21 \\ \hline 0 &= 0, \text{ true} \end{aligned}$$

The true statement indicates that the system has infinitely many solutions.

Solution Set: $\{(x,y) | x - 5y = 7\}$ or $\{(x,y) | 3x - 15y = 21\}$

Objective 3: Determine the most efficient method for solving a linear system.

Comparing Solution Methods		
Method	Advantages	Disadvantages
Substitution	Gives exact solutions. Easy to use *if a variable is on one side by itself*.	Solutions cannot be seen. Introduces extensive work with fractions when no variable has a coefficient of 1 or -1.

Objective 3: Determine the most efficient method for solving a linear system.

Comparing Solution Methods		
Method	Advantages	Disadvantages
Addition	Gives exact solutions. Easy to use *if a variable has a coefficient of 1 or -1.*	Solutions cannot be seen.

Objective 3: Determine the most efficient method for solving a linear system.

Comparing Solution Methods		
Method	Advantages	Disadvantages
Graphing	You can see the solutions.	If the solutions do not involve integers or are too large to be seen on the graph, it's impossible to tell exactly what the solutions are.

OBJECTIVES:

1. Solve linear systems by the addition method. ✓
2. Use the addition method to identify systems with no solution or infinitely many solutions. ✓
3. Determine the most efficient method for solving a linear system. ✓